Fluid Mechanics
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Lecture – 19
Dimensional Homogeneity

Welcome all of you to this lectures on dimensional analysis. This very interesting lectures what today I will cover it talking about how to do the experiments in fluid mechanics to characterize the probable. So very interesting topic and this topic I will cover it in 2 lectures.

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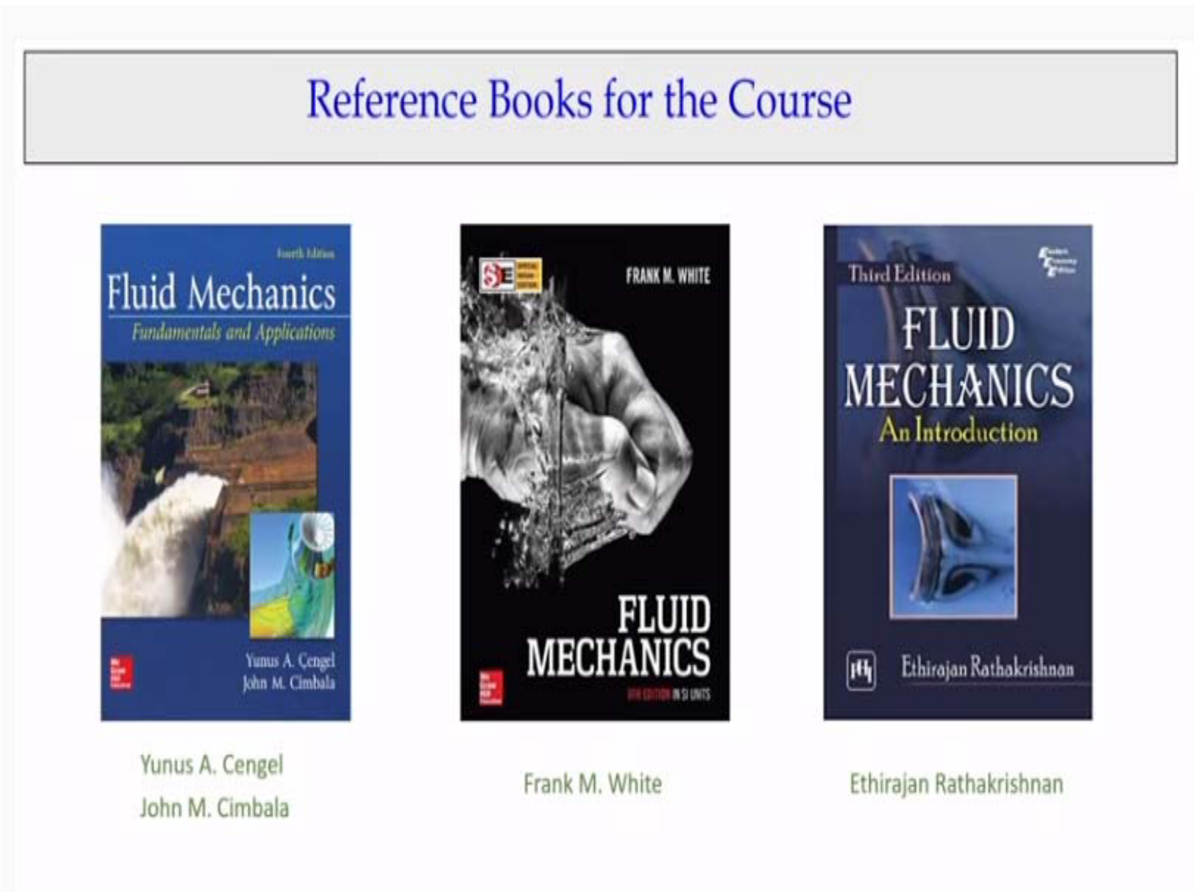

So today lectures and the next lectures what I will cover it? Again I am following these 3 books which already we discussed earlier Cengel, Cimbala, FM.White and Ethirajan Radhakrishnan.

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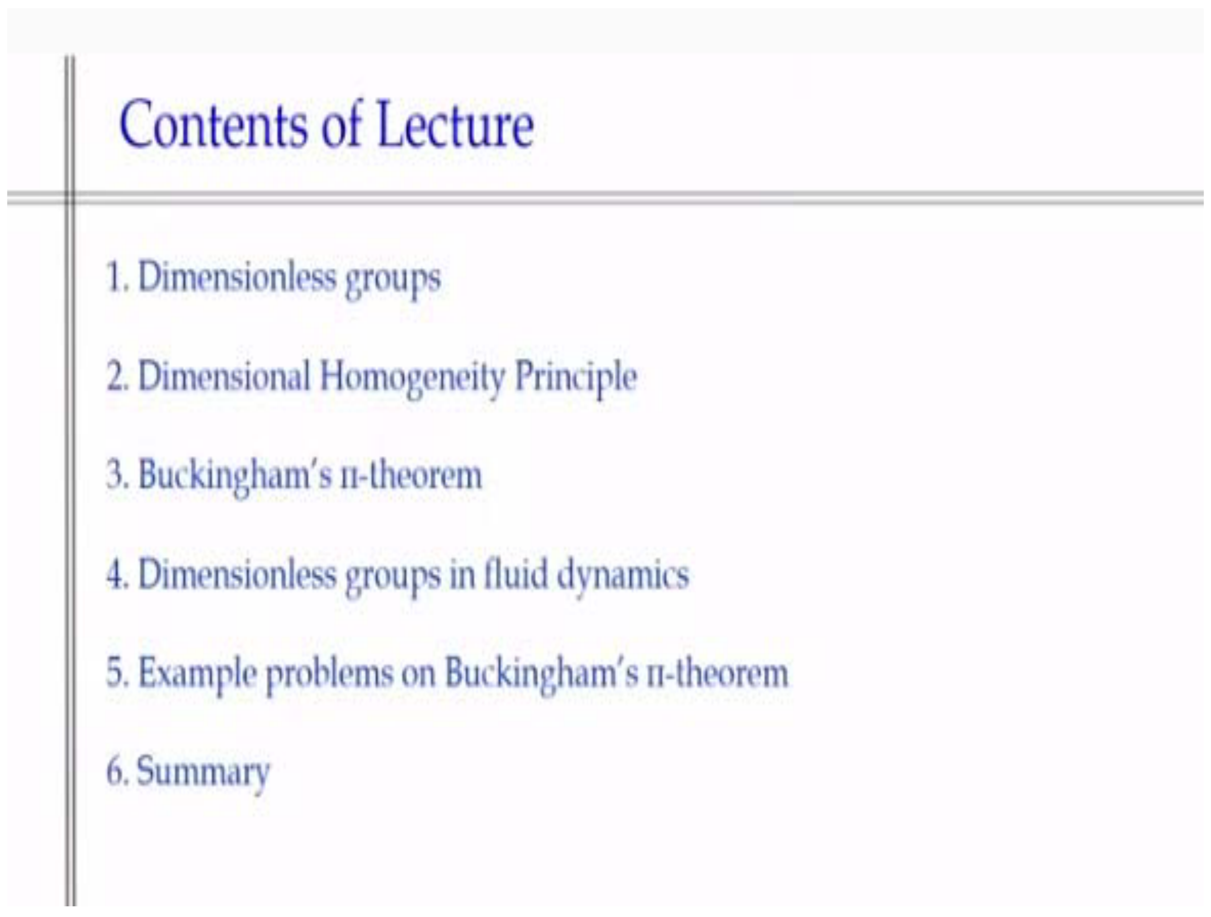

So today I will start with lots of good examples what we have gained? And we discussed that so I will talk about Dimensionless groups, Dimensional Homogeneity principles, Buckingham's pi theorem, Dimensionless groups what is used in fluid flow that is what I will introduce to you then we will discuss some example problems based on Buckingham's-π theorem and we will have a summary. So as I stated earlier let us start with very interesting examples.

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What we observed in May 3rd, 2019 you can see this these overturning bus overturning in these because of the super cyclonic what it happened in May 3rd, 2019 and this is what we conducted the experiment in wind tunnel in IIT Guwahati. The question rise that as you move it is a full scale models okay prototype and this is a model this is a full scale that is what it got it in the cyclonic storm centre this is what I conducted in it a wind tunnels and we are getting the similar trend.

Are they enough for a study to conduct or we need to do some sort of a similarity analysis the flow pattern analysis how do we do it? How do we design these experiments? This is what I am just demonstrating when you conduct the fluid experiments you have to first design the experiment how we do that. So those things I will today I will discuss it before that.

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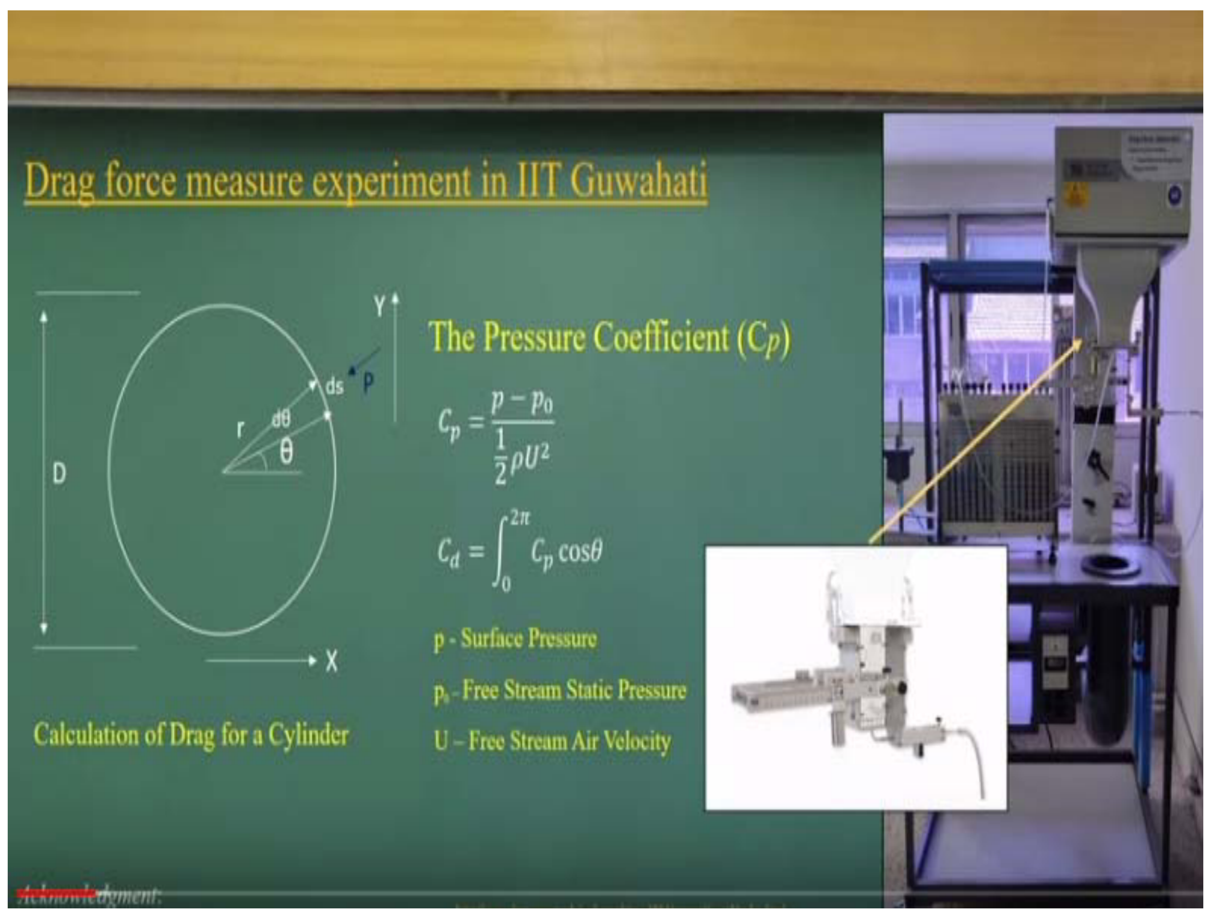

I am to just show you that there was an instrument which can measure the drag force measurement experiment set up like these where you can compute the pressure co efficient and if you integrate it in get the drag co efficient and that way when you have put a cylinders you can compute the what could be the drag the force acting on these cylinders with the varying the fluid flow the air flow conditions here.

The Pressure Coefficient (Cp)

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho U^2}$$

$$C_d = \int_0^{2\pi} C_p \cos\theta$$

p - Surface Pressure

p_0 – Free Stream Static Pressure

U – Free Stream Air Velocity

So these type of experiments this is a possible now and we also have the facilities in department of Kinetic energy to visualize that how the drag force are hafting it one case I showed you the wind tunnel and other so case I have so neat that how you can measure the drag force measurement in IIT Guwahati set up what we have where you can measure the drag force okay as you know drag force is a very preliminary data for any experimental designing. So drag force we use for these case we are just talking about drag force for a cylinder.

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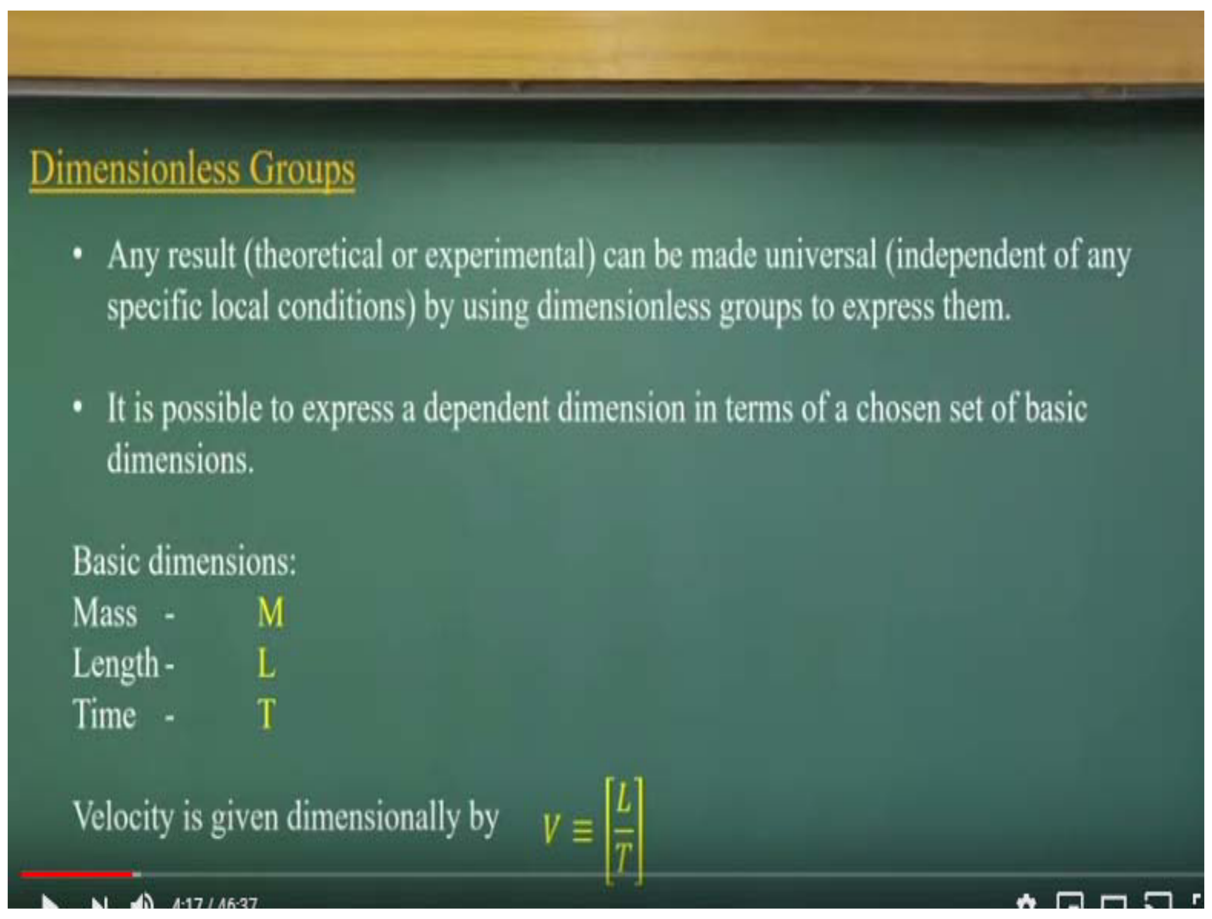

Now we will come to the dimensionless groups I think these quite you know it. Even at the class 11th or 12th levels. So basic dimensions what we have mass length and the time these are 3 basic dimensions mass, length and time. So what we have when we would do a theoretic analysis or the experiment analysis can be made universal okay that is what exactly highlighting here independent of its specific locations using a diverse group to express them.

What I am talking is that we will discuss more in these we conduct the experiment but the experiment becomes inverse now when you make it a dimensionless group analysis. Otherwise it will be a particular case studies not the experiments what were we innovation study. So we have a very basic dimensions of any variables that mass length and the time as you know it what is the velocity? The distance or in the time? The length by time so any of the fluid flow variables we can define in terms of these 3 basic dimensions mass length and time.

Basic dimensions:

Mass - M

Length - L

Time - T

Velocity is given dimensionally by $V \equiv \left[\frac{L}{T}\right]$

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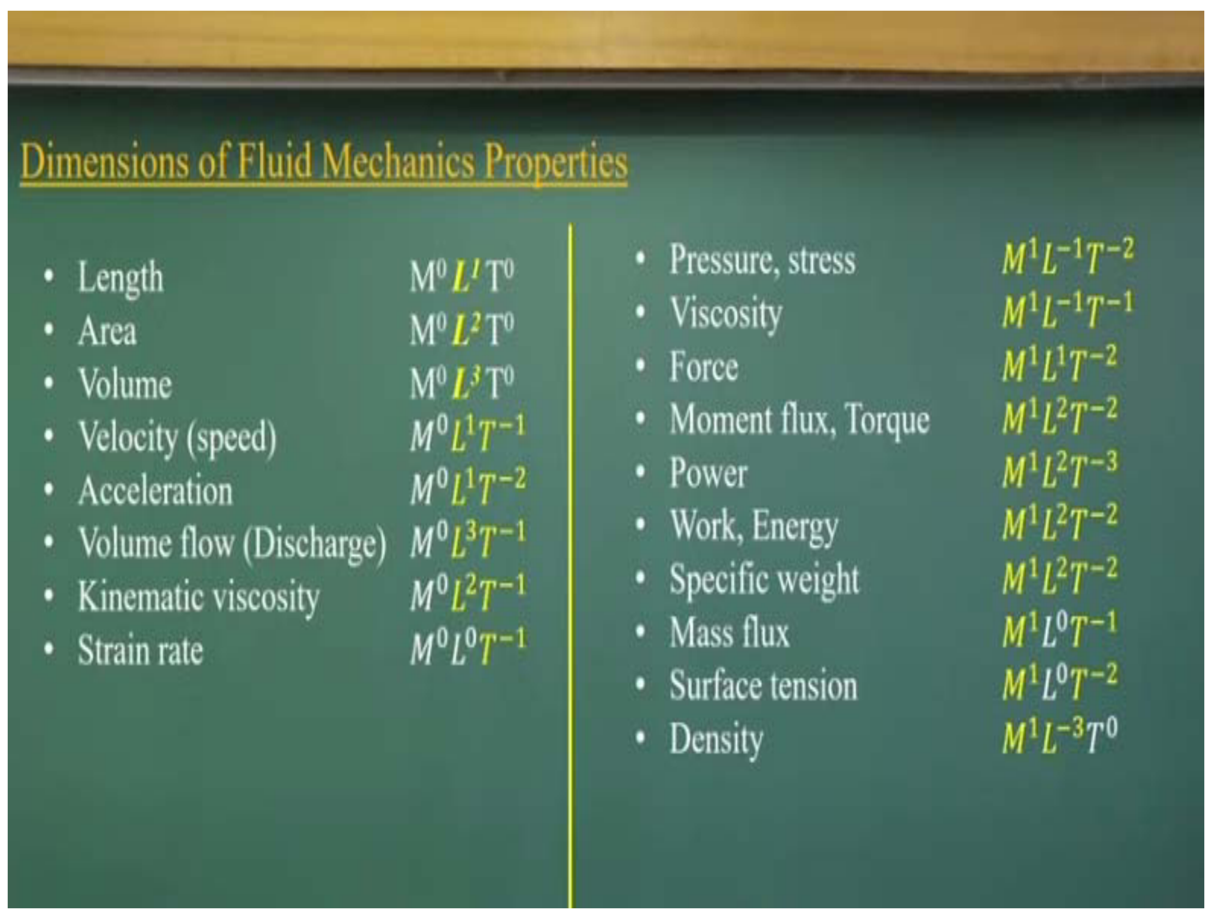

Now if you look at the fluid properties what we have some of the fluid properties if you know it is related to the dimensions okay that is the length area and the volume. So it is just a dimension and geometric dimensions so it has only unit in terms of the length okay. So that means length is L^1 L^2 and area will be L^2 the volume will be L^3 so you can understand what is the velocity you know it length power unit time.

The acceleration similar way you can do it let me look at what is the distance or volumetric rate volume or unit time that is what we Lq divide by That will be the volumetric then now we have come to the 2 different parts one is Kinematic viscosity as we discuss in newtons laws of viscosity is that it is independent of mass. So you can understand it has a dimensions of length and time.

Similar way we see a Strain rate also indifferent of mass and length. So if you look at these properties all are independent to mass. First I start from length area, volume then fluid properties like velocity, acceleration volumetric rate or the discharge kinematic viscosity and the strain rate. Now if you look at other part like pressures force per unit area similarly case force per unit area. So force you know it that will be the mass into acceleration the force will be mass into acceleration the pressure and stress will be the force per unit area.

That is what you can compute it. Similar way you can compute the viscosity dimensions if you know very basic equation of Newtons law of viscosity. Just Shear stress rate is proportional to Shear strain rate proportionality is the viscosity. So you can compute it what could be the dimensions of the viscosity. Similar way let us come to next levels with the momentum flux other torque that what we will have this unit the power, work and energy you can find out what will be the unit then I will be coming to the other 2 properties is specific weight and Mass flux.

Mass per unit time that is what is mass flux similar way the surface tension you can compute what will be the surface tension so that one is force per unit length. That is if we look at that force per unit length is really come into its surface tension. The density mass per unit volume so if you look it if you know very basic definitions of these fluid properties you can easily write these properties in terms of dimensions.

In terms of mass length and the time so it is very easy if are having any doubt over that you take a equations like you have a doubt about what could be the dimensions of viscosity then you will write newtons law of viscosities that is what tau shear stress is equal to mu dv/dy substitute the shear stress what is the unit dv/dy what is the unit then you will get the mu because this is the dimensionless homogeneous equation.

So what I am to talk about that if you are just forgot that what could be the dimensions of a particular fluid properties try to remember a dimensionless homogeneous equations with that properties are involved if you can get it that and substitute the dimensions of the properties like here this shear stress the velocity gradient and the μ value then you can get it what will be the dimensions of mu and you will know it what is the relationship between the kinematic viscosity and the viscosity or the dynamic viscosity.

So if you know the basic relationship between the fluid properties also we can compute it what will be the dimensions of the fluid properties. So whenever you have the problems first you write the dimensions of the fluid properties and if you know the dimensions of fluid properties then you can easily find out what could be its unit okay in terms of kg per meter cube what is the unit of that what you can make it mass will be in kg length will be in meter or centimetres.

And the time will be in second, hour, day we can have a different timeframe. So it is a very easy task to make it a dimensions of the fluid mechanics properties.

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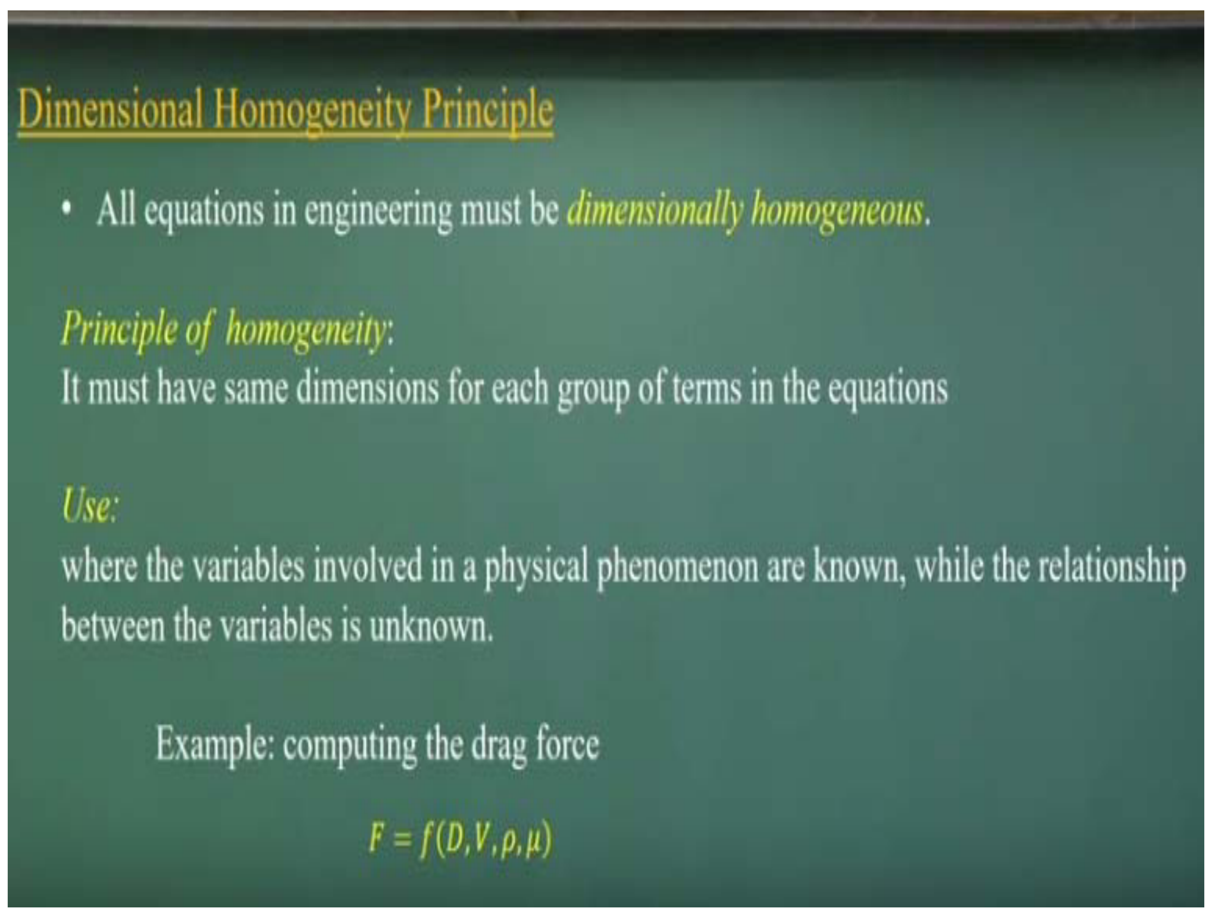

Let us go for what is the principal of homogeneity all the equations not all the equations so most of the questions in engineering the most of the equations of engineering dimensionally homogenous not all that is the let me have a repeat these things that means what it indicates as that the dimensions of the equations will be the same okay the left side of dimensions LHS should have a dimensions of right hand side.

Then the equations are dimensionally homogeneous so that means what do we have to look at that for any physical political properties is and all so somewhere it follows this dimensional homogeneous concept. Those are concept we have used it for designing the experiment for example if I compute a drag force like we have the experiment set up what I showed it that I have a cylinders I am making a velocity V and I try to know it what will be the drag force.

Okay if it is that kind of conditions if that is the condition if is the velocity V these the diameters are the cylinders and the drag force f_D.

$$F = f(D, V, \rho, \mu)$$

I am looking it so other 2 properties also matters it ρ and the μ is the viscosity. So it depends upon the viscosity that means it depends upon whether you have an air or the water so it density varies the viscosity varies. So the drag force what is happening is it is a function of D is the diameter of the cylinder V is the velocity of the flow.

And the rho and the mu is the fluid properties related to density and the dynamic viscosities the viscous force components it depends upon the like you can know it the drag force court in oil will be the different compared to drag force in air. So that with the mu will take care of what type of drag force will be there D also depends if a bigger diameter or smaller diameter D also matters.

So the drag force is a function which we do not know it okay D is the functions of the diameters V velocity of the flow and the density and dynamic viscosity if we start now I have to design the experiment to finding out this F component how do we do it there is 2 way to do.

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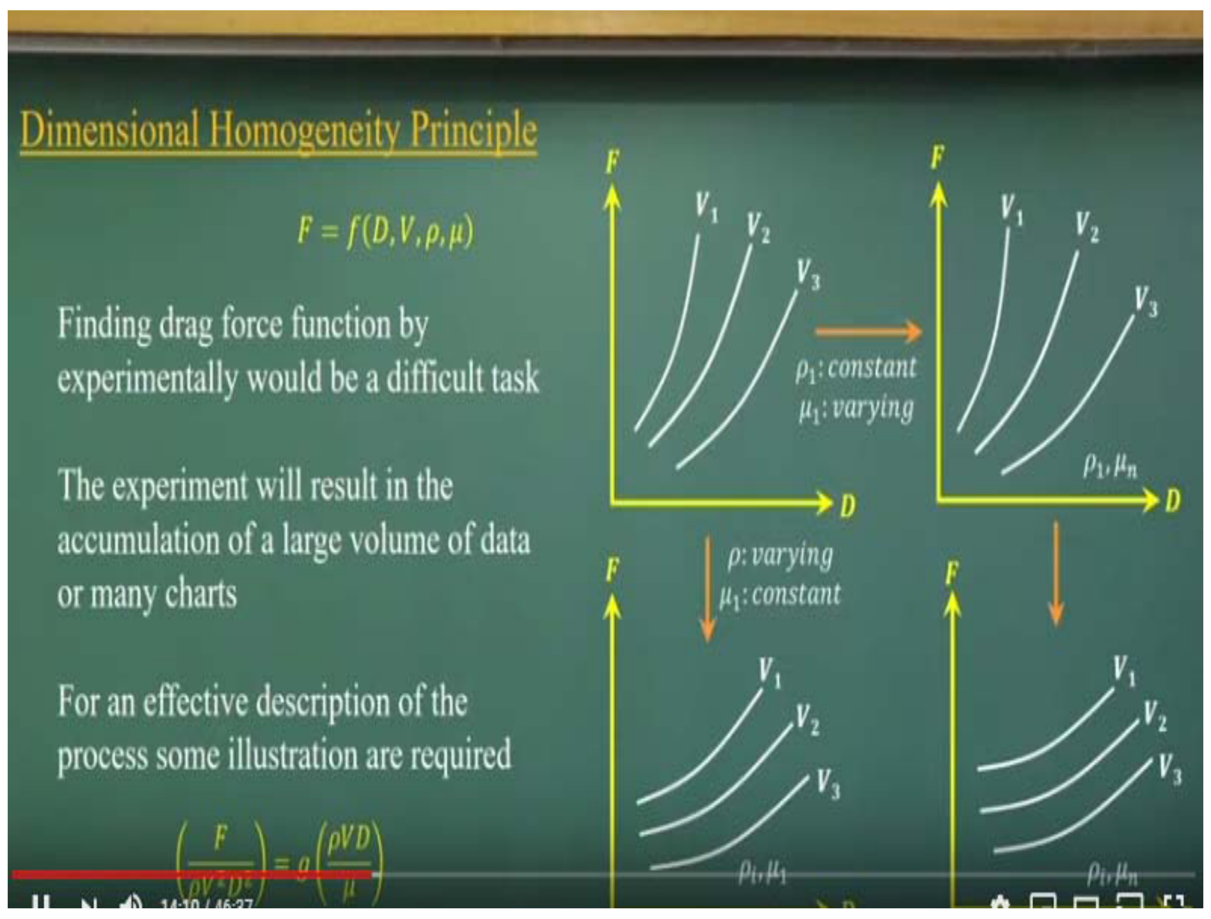

First is that I know it what I can do it since F is a function we get 4 independent variables I will do a 4 type of experiments that means when I am conducting an experiment between changing the D and different V_1, V_2, V_3. I have a making a constants of ρ and μ. Similar way I will change the other parameters keeping other parameters constants. See if I am trying to do that because how to try to get a functions on that I will give it 2 parameters fixed and I will run for a series of experiment.

Let be a 10 experiment I will do it to get a curve if that is the conditions that means for the 3 independent variables I will do it 10 into 10 into 10 into 10 that is what will come out to be the 1000 experiment. So if it is a 1000 experiment then it is too expensive for us but what we have to look at that? Is there any some sort of dimensions relationship is there between this independent variables and the dependent variable if they have the dimensional relation and that relationship you can get it we need not to do this 1000 experiments.

We can do less number of experiment that means you can do a 10 experiment if you do a dimensional analysis to design these experiments you just do a 10 experiment to complete this process what we have to do it we make a non-dimensional curve. That $\frac{F}{\rho V^2 D^2}$ if you just substitute the dimensions you will see that it does not have a dimension of this $g\left(\frac{\rho V D}{\mu}\right)$ also have a does not have a dimensions.

$$\left(\frac{F}{\rho V^2 D^2}\right) = g\left(\frac{\rho V D}{\mu}\right)$$

You just substitute it and try to find out $g\left(\frac{\rho V D}{\mu}\right)$ which is very well known equations which we call the Reynolds equations. So okay Reynolds numbers okay we will discuss more about that in pipe flow and so if we look at that what I developed now I instead of F functions we are looking at G functions which is a really sensitive tool non-dimensional group. One is left side and the right side this would have the same and I will be looking at different functions if it that did the conditions

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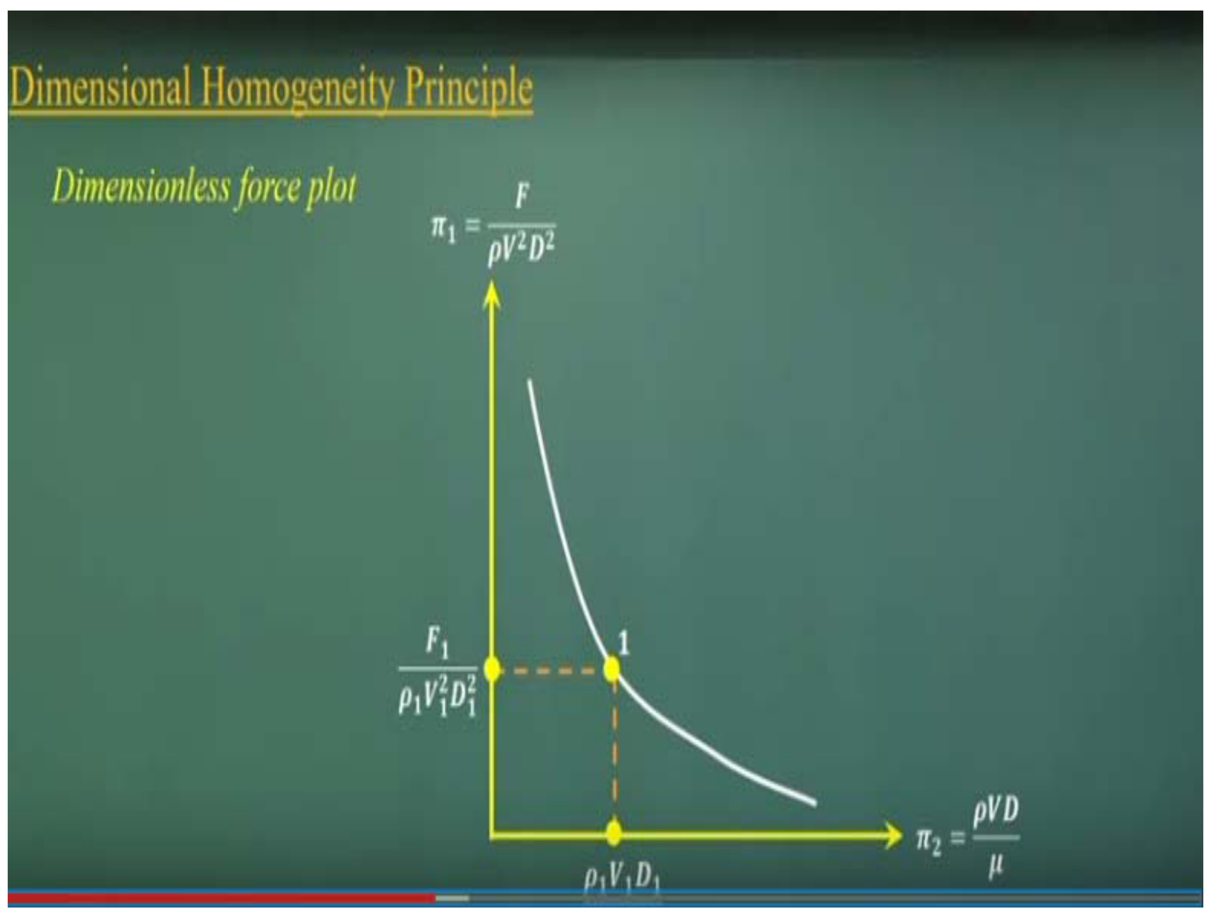

I should conduct a relationship of experiment of different Reynolds numbers and can get it from the experiment but we need the force and non dimensionless as a F_1 functions and void heat wave functions getting these. So only 10 experiment are enough for this study as compared to conducting the 1000 experiment. So the non dimensionless experiments making us that we can make a dimensionless force plot.

And we can conduct the experiment with the different Reynolds numbers and plot because once you do with different Reynolds experiment you will get the course you know the velocity you move the diameters of the cylinders then you can compute it what will be the non dimensionless force component here and if you get this curve then it is easy to find out at different Reynolds numbers what will in the force.

So one is that these dimensionless analysis is saves us though doing a large number of experiments second is that when you put a dimensionless force it easy to interpret to the data as compared to the doing the individual basis. So the dimension analysis plot and this analysis help us to save the large number of experiments as well as this it help us to interpret it data better way as compared to do individuals.

$$\pi_1 = \frac{F}{\rho V^2 D^2}$$

$$\pi_2 = \frac{\rho V D}{\mu}$$

Almost all the time do any experimentalists they start designing this experiment using dimensional analysis. It is not only the fluid mechanics economic models socio economic models you will talk about any experiment what you conduct it we always do a dimensional analysis to find out the how many experiments we should conduct.

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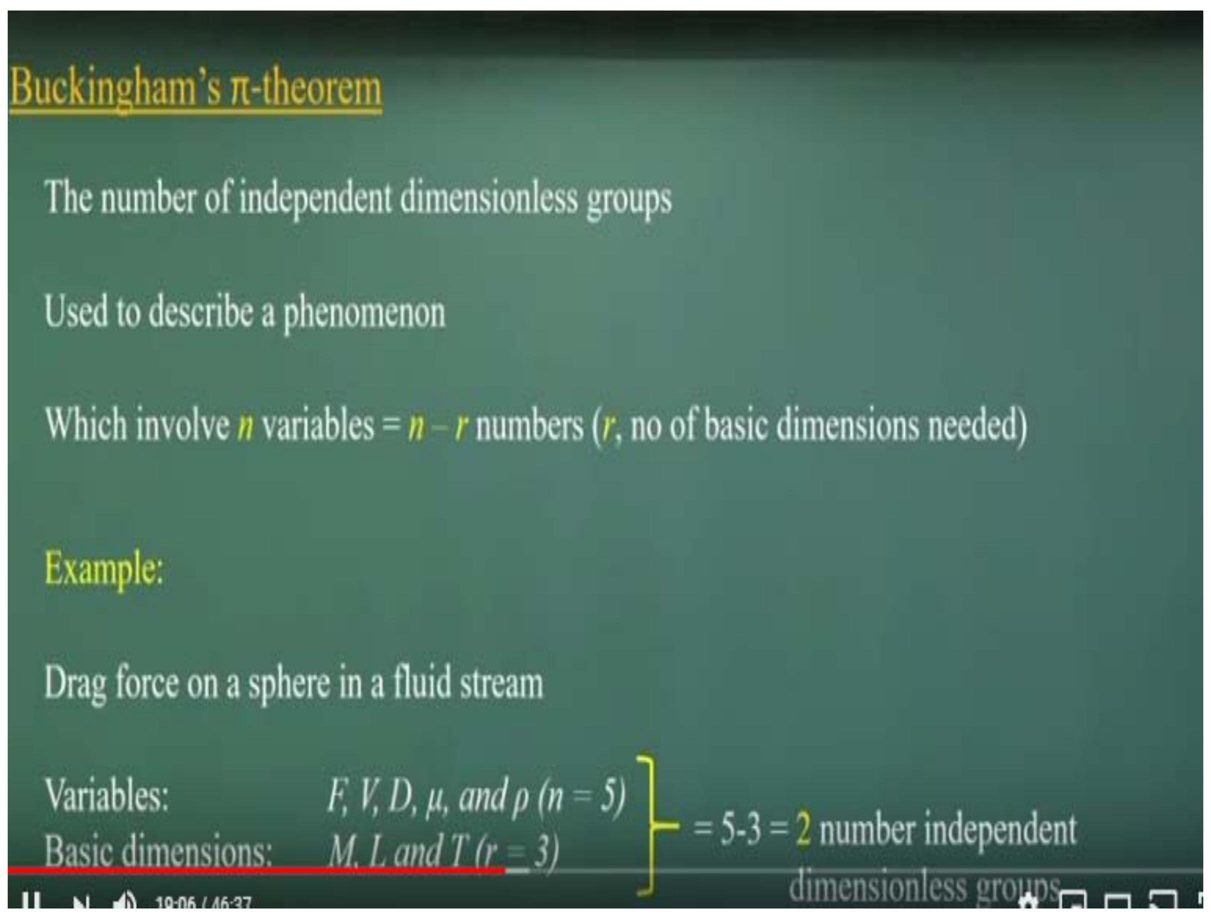

Mow let us commit how to do that is Buckingham's π −theorem it is a very simple π −theorem concept is that so we will have a number of independent dimensionless groups so how many number of independent dimensionless groups I look at it if there is a n dependent variables okay we know it basic dimensions are 3 mass length and the time so that means we can group it a non dimensionless number will be the n-r please remember as you will know it the dimensions will be 3 but not always be 3 okay?

That is again I highlighted right I need to have a judgment for that if you know that the particular defendant variables have it its having these many of dependent variables if you can visualize that what are the force component? What about the variables are going to play for that components then you will do a non-dimensional analysis to find out the real relationship between dependent and independent variable?

Now let us come back to that same problems if a drag force on a sphere okay let us see easy now okay same concept in a fluid streams the same problems okay. You will have a sphere here and you have the flow is coming here like a cricket balls okay? You are throwing the cricket balls okay and you would try to find out that what could we the drag force okay what could be the drag force is acting on that.

What will be the drag force acting in that so if that is the conditions I can visualize that it has a dependency of velocity the dimensions of this the diameter of the sphere the fluid properties like mu and the rho. So the total number of dependent and independent variables are 5 basic dimensions are 3. So we can make 2 number of independent dimensionless groups is correct its very easy things that we have to find out the defendant variables and the indifferent variables count that and we know the basic dimensions are the 3 mass length and the time and we just subtract it and find out that we can have a 2 number of independent dimensional group.

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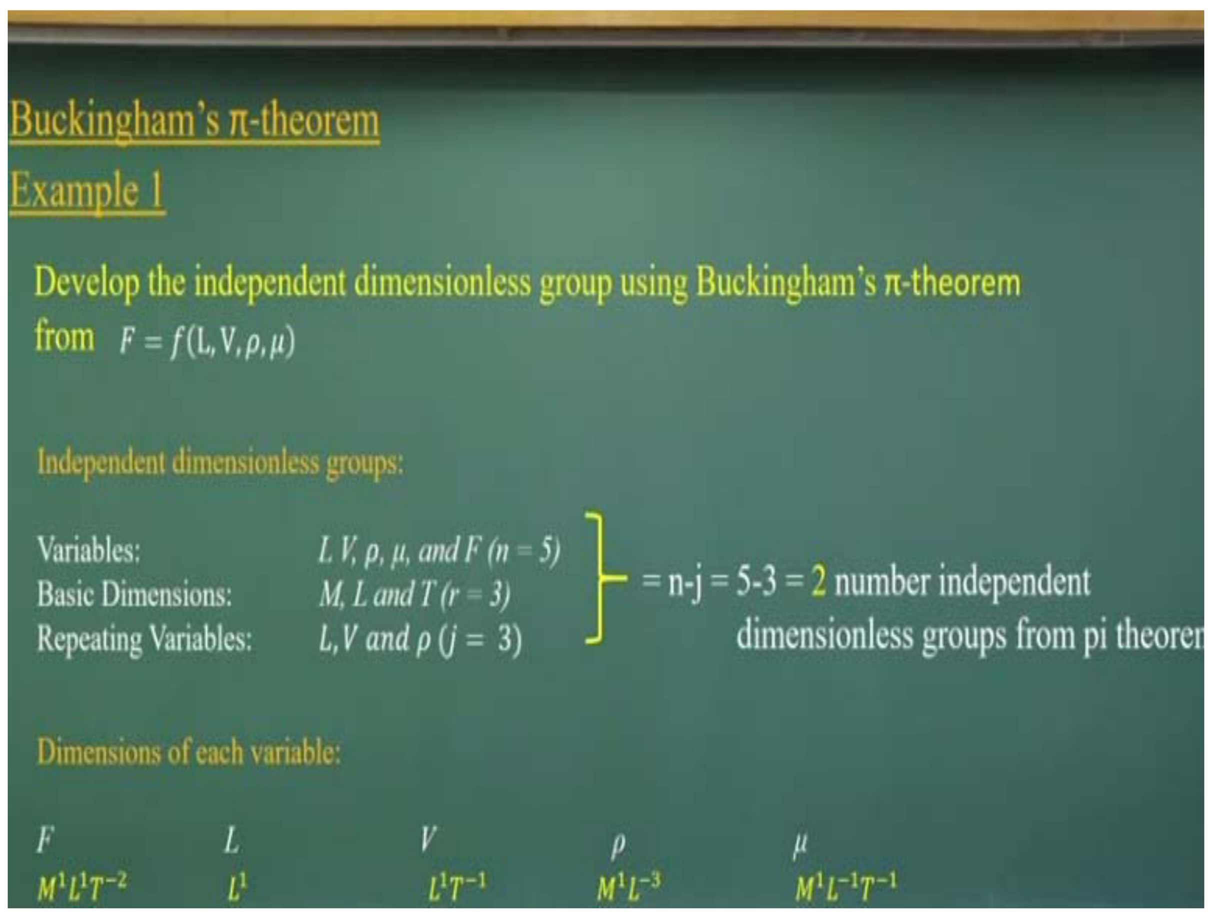

Now I have to develop the dimensional groups of that so if you look at that way first what I have to do it I have to write the dimensions of each force, Mass into excellence the length the velocity density and dynamic viscosity. If you do not remember that dimensionless viscosity the dimensionless please remember newton's law of viscosities okay? Anyone you have to remember it okay so we put do not remember Newton's law of dynamic viscosity please remember what newton's laws of viscosity.

If you remember it, you can originally compute it what could be the mu dimensions that is what always you can prove any of the exam that is I want to give you that in order to remember all the dimensions of the fluid properties but you should know the Basic equations some of the basic equations you can easily find out what could be the dimensions of that depending on independent variable.

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